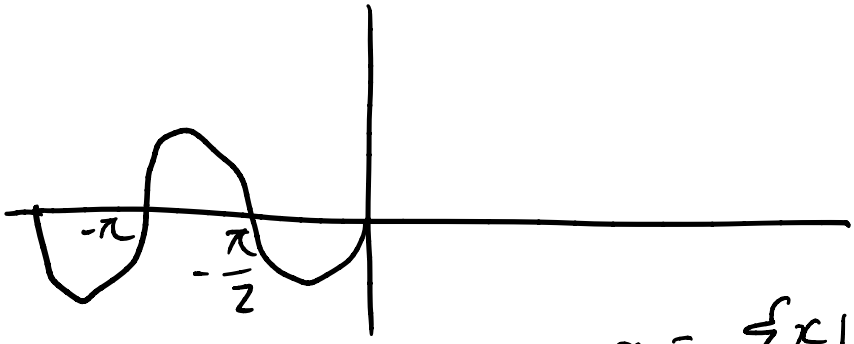
6/1/25

Continuous but not Smooth

Find values of the constants a and b for which the following function is continuous but *not* differentiable.

$$f(x) = \begin{cases} ax + b, & x > 0; \\ \sin 2x, & x \leq 0. \end{cases}$$

In other words, the graph of the function should have a sharp corner at the pont $(0, f(0))$.



$a = \{x \mid x \neq 2\}$

$b = 0$

$f_1(x) = ax + b$

$f_2(x) = \sin 2x$

$f_1'(x) = a$

$f_2''(x) = 2\cos 2x$

$$\lim_{x \to 0} f_2'(x)$$

$$= \lim_{x \to 0} 2\cos 2x$$

$$= 2\cos 2(0)$$

$$= 2$$